SHANKLEA PRIMARY SCHOOL

Written Calculation Policy

Policy Control Details			
Date policy approved:	September 2016		
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Approved for issue by:	Gareth Pearson	Signature	Date
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Shanklea Primary School Written Calculation Policy

Progression towards a standard written method of calculation

Introduction

This calculation policy has been written in line with the programme of study taken from the revised **National Curriculum for Mathematics (2014).** It provides guidance on appropriate calculation methods and progression. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division.

Statements taken directly from the programme of study are listed in bold at the beginning of each section.

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

Aims of the Policy

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence an understanding

How to Use this policy

- Use this policy as the basis for your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always us Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- If, at any time, children are making significant progress, look ahead to the next stage in calculation
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems

Stages in Addition

Addition – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to **combining two groups of objects**, first by **counting all** and then by counting on from the largest number.

They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

'You have five apples and I have three apples. How many apples are there altogether?'

Addition – Year One

- **Given a number, identify one more**
- **Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign**
- **Add one-digit and two-digit numbers within 20, including zero**
- **Solve missing number problems e.g. 10 + ☐ = 16**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practice counting on from any number e.g. 'Put five in your head and count on four.'

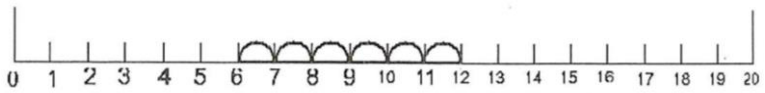
Initially use a **number track** to count on for addition, counting on from the largest number:

5 + 4 = 9

'Put your finger on number five. Count on (count forwards) four.'

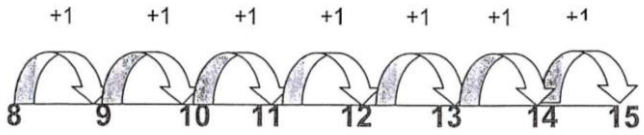
Then progress to a **marked number line:**

6 + 6 = 12



'Put your finger on number six and count on six.'

8 + 7 = 15 'Put your finger on number eight and count on seven.'



Ensure children are confident with using a marked number line before moving on to an empty number line (see Year Two guidance).

Continue to practice counting on from the largest number for addition with totals within 20.

Addition – Year Two

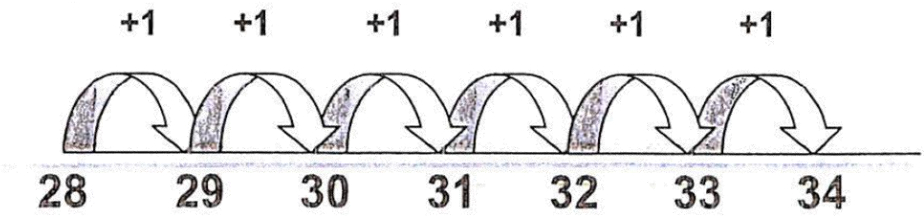
Add numbers using concrete objects, pictorial representations, and mentally, including:

- **A two-digit number and ones**
- **A two-digit number and tens**
- **Two two-digit numbers**
- **Three one-digit numbers**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

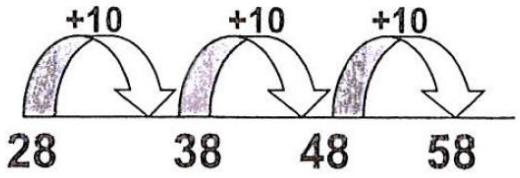
Counting on in ones using an **empty number line**, within 100….

28 + 6 = 34



…. And in tens

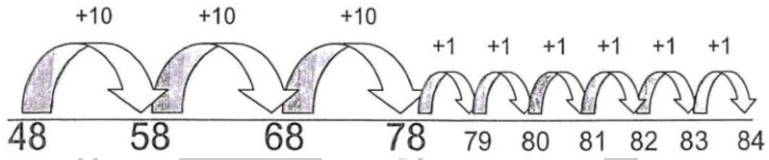
28 + 30 = 58



Use in conjunction with a **100 square** to show jumps of tens.

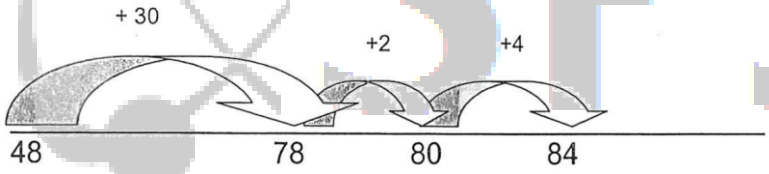
48 + 36 = 84

'Put the biggest number first (48), and then partition the smaller number (36 = 30 + 6) and count on: 48 + 30 + 6.'



Use in conjunction with a **100 square** to show jumps of tens and ones.

If children are confident, use more efficient jumps…



Use in conjunction with a **100 square** to show jumps of tens and ones/units.

Also use the **partitioning method** to add two two-digit numbers:

43 + 25 = 68

40 3 20 5

40 + 20 = 60
3 + 5 = 8
60 + 8 = 68

‘Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together.
Recombine to give the answer.’

Then move on to calculations that **bridge** the tens:

48 + 36 = 40 + 8 + 30 + 6

40 + 30 = 70
8+ 6 = 14
70 + 14 = 84

48 + 36 = 84

This is an alternative way of recording the partitioning method.

Further develop addition with numbers that bridge 100, using a **200 grid** to support.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

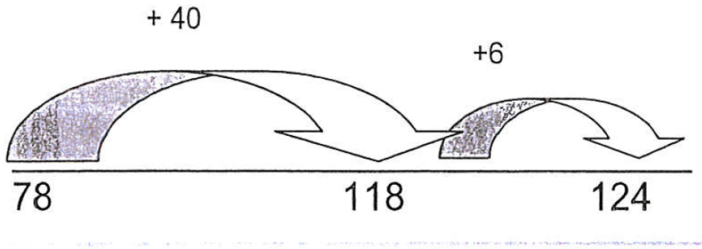
Addition – Year Three

- **Add numbers with up to three digits, using formal written method of columnar addition.**

N.B. Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

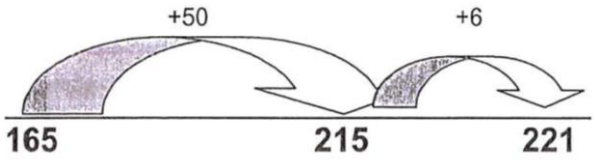
Further develop the use of the **empty number line** with calculations that **bridge 100**:

78 + 46 = 124



Use a **200 grid** to support counting on in tens and bridging 100….
… and with addition of a three-digit and a two-digit number:

165 + 56 = 221



Further develop the **partitioning method** with calculations that **bridge 100:**

85 + 37 = 80 + 5 + 30 + 7

80 + 30 = 110
5 + 7 = 12
110 + 12 = 122
85 + 37 = 122

The partitioning method can also be used with three-digit numbers.

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it has not been necessary to bridge across the tens or hundreds:

63 + 32 = 95

$$\begin{array}{r} 60 + 3 \\ +\ 30 + 2 \\ \hline 90 + 5 = 95 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then….

$$\begin{array}{rll} & 6\,3 & \\ +\ & 3\,2 & \\ \hline & 5 & (3 + 2) \\ +\ & 9\,0 & (60 + 30) \\ \hline & 9\,5 & \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

This will lead into the formal written method…

$$\begin{array}{r} 63 \\ +\ 32 \\ \hline 95 \end{array}$$

Use the language of place value to ensure understanding 'Three add two equals five. Write five in the units column. 60 add 30 equals 90. Write 9 (90) in the tens column.

N.B. Informal/mental methods would be more appropriate for numbers of this size, but use two-digit numbers when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an **expanded method** initially:

68 + 24 = 92

$$\begin{array}{r} 60 + 8 \\ +\ 20 + 4 \\ \hline 80 + 12 = 92 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then….

$$\begin{array}{rll} & 68 & \\ + & 24 & \\ & 12 & (8 + 4) \\ + & 80 & (60 + 20) \\ \hline & 92 & \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready, introduce the **formal written method**, where it is necessary to 'carry' ten from the units to the tens column:

$$\begin{array}{r} 68 \\ +\ 2\overset{1}{,}4 \\ \hline 92 \end{array}$$

Use the language of place value to ensure understanding 'Eight add four equals twelve. Write two in the units column and 'carry' one (10) across into the tens column. 60 add 20 and the ten that we 'carried' equals 90. Write 9 (90) in the tens column. 92 is the answer

The digit that has been 'carried' should be recorded above the line in the correct column.

When children are confident, extend with examples where it is necessary to bridge across the tens and the hundreds:

76 + 47 = 123

$$\begin{array}{r} 70 + 6 \\ + \ 40 + 7 \\ \hline 110 + 13 = 123 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then….

$$\begin{array}{rl} & 76 \\ + & 47 \\ \hline & 13 \quad (7 + 6) \\ + & 110 \quad (70 + 40) \\ \hline & 123 \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready, introduce the **formal written method**, where it is necessary to 'carry' across the columns and bridge 100:

76 + 47 = 123

$$\begin{array}{r} 47 \\ + \ 7_{1}6 \\ \hline 123 \end{array}$$

Use the language of place value to ensure understanding 'Seven add six equals thirteen. Write three in the units column and 'carry' one (10) across into the tens column. 40 add 70 and the ten that we 'carried' equals 120. Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).

The digits that have been 'carried' should be recorded above the line in the correct column.

If children are confident, further develop with the addition of a three-digit number and a two-digit number:

178 + 43 = 221

$$\begin{array}{r} 178 \\ + \ 43 \\ \hline 221 \end{array}$$

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year Four

- **Add numbers with up to 4 digits, using formal written method of columnar addition.**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Further develop the formal written method of addition, with three-digit numbers. Revisit the **expanded method** first, if necessary.

176 + 147 = 323

$$\begin{array}{rll} 176 & & \\ +\ 147 & & \\ \hline 13 & & (7 + 6) \\ +\ 110 & & (70 + 40) \\ 200 & & (100 + 100) \\ \hline 323 & & \end{array}$$

This will lead into the **formal written method**..

176 + 147 = 323

$$\begin{array}{r} 147 \\ +\ {}^{1}1{}^{1}76 \\ \hline 323 \end{array}$$

Use the language of place value to ensure understanding 'Seven add six equals thirteen. Write three in the units column and 'carry' one (10) across into the tens column. 40 add 70 and the ten that we 'carried' equals 120. Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100). 100 add 100 and the 100 that has been carried equals 300. Write 3 in the hundreds column (300).

The digits that have been 'carried' should be recorded above the line in the correct column.

If children are confident, introduce the addition of a four-digit number and a three-digit number:

1845 + 526 = 2371

$$\begin{array}{r} 1845 \\ +\ {}_{1}5{}_{1}26 \\ \hline 2371 \end{array}$$

Continue to develop with addition of two four-digit numbers and with decimals (in the context of money or measures).

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition – Year Five

- **Add whole numbers with more than 4 digits, including using formal written method (columnar addition)**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers (and decimals), as appropriate.

Continue to develop the **formal written method for addition** with larger numbers (and decimal numbers) and with the addition of three or more numbers.

21848 + 1523 = 23371

$$\begin{array}{r} 21848 \\ +\ 1523 \\ \hline 23371 \end{array}$$

Continue to use the language of place value to ensure understanding. Ensure the digits that have been 'carried' are recorded above the line in the correct column.

Use the **formal written method** for the addition of decimal numbers:

£154.75 + £233.82 = £388.57

$$\begin{array}{r} 154{\cdot}75 \\ +\ 233{\cdot}82 \\ \hline 388{\cdot}57 \end{array}$$

Continue to use the language of place value to ensure understanding.

Ensure that the decimal points line up.

Continue to practice and apply the formal written method throughout Year 5.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition – Year Six

No objectives have been included in the programmes of study explicitly related to written methods for addition in Year 6. However, there is an expectation that children will continue to practise and use the **formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous year's guidance for methods).

Our aim is that by the end of Year 6, children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use and efficient **formal written method** accurately and with confidence.

Stages in Subtraction

Subtraction – Early Stages (EYFS)

Children will engage in a variety of counting songs and thymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' **using objects** to count 'how many are left' after some have been taken away.

6 – 2 = 4

'Take two apples away. How many are left?'

Children will begin to count back from a given number.

Subtraction – Year One

- **Given a number, identify one less**
- **Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign**
- **Subtract one-digit and two-digit numbers within 20, including zero**
- **Solve missing number problems eg 20 -☐ = 15**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practise counting back from a given number.

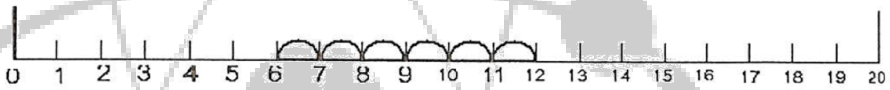
Initially use a **number track** to **count back** for subtraction:

9 – 5 = 4

'Put your finger on the number nine. Count back five.'

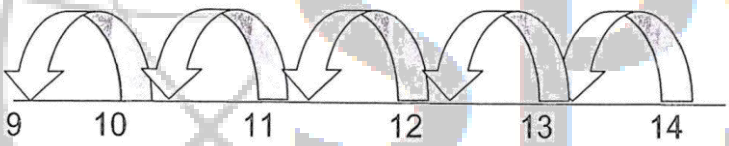
Then progress to a **marked number line:**

12 – 6 = 6



'Put your finger on the number twelve and count back six.'

14 – 5 = 9



'Put your finger on number 14 and count back five.'

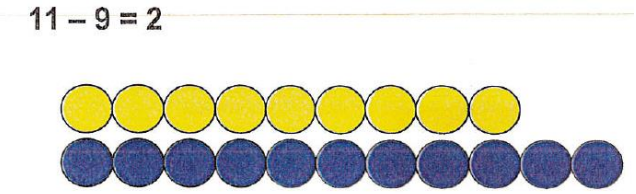
N.B. Ensure that children are confident with a **marked number line** before moving on to an empty number line (see Year Two guidance).

Continue to practise counting back for subtraction with numbers within 20.

Counting on to find a small difference:

Introduce complementary addition to find differences (only use for **small** differences). The use of models is extremely important here to understand the idea of "difference."

Count up from the smallest number to the largest to **find the difference** using resources, e.g. cubes, beads, number tracks/lines:



The **difference between** nine and eleven is two.

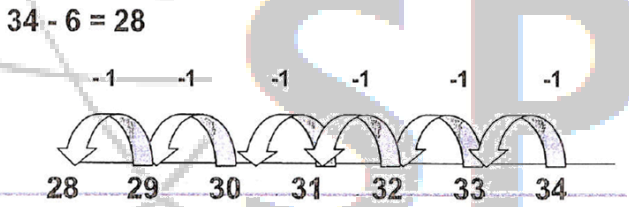
N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtraction – Year Two

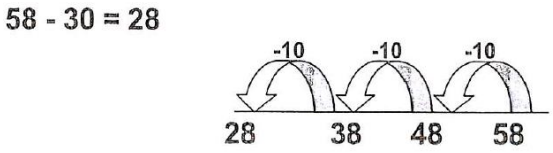
- **Subtract numbers using concrete objects, pictorial representations, and mentally, including:**
    - **A two digit number and ones**
    - **A two digit number and tens**
    - **Two two-digit numbers**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Counting back using an **empty number line** within 100, in ones….

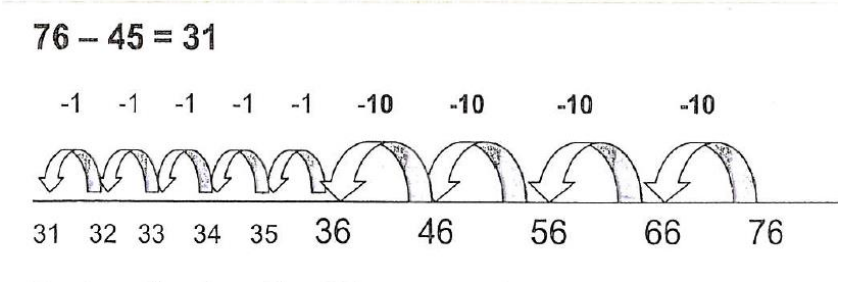


And in tens:



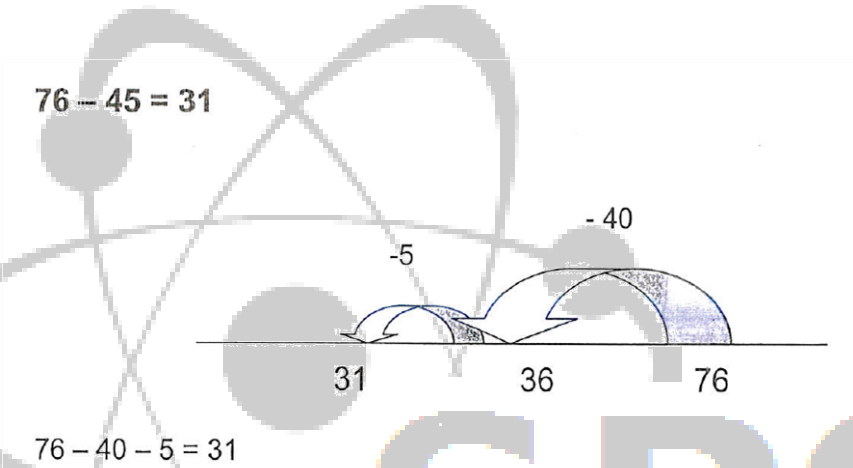
Use in conjunction with a **100 square** to show jumps of tens.

Subtraction, using partitioning, on an empty number line:



Use in conjunction with a **100 square** to show jumps of tens and ones.

If children are confident, use more efficient jumps:



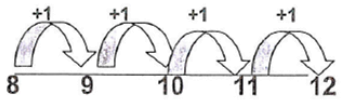
Use in conjunction with a **100 square** to show jumps of tens and ones.

Counting on to find a small difference

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important to understand the idea of "difference" (See Year 1 guidance)

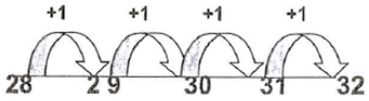
Count up from the smallest number to the largest to **find the difference**.

12 - 8 = 4



'The difference between 8 and 12 is 4.'

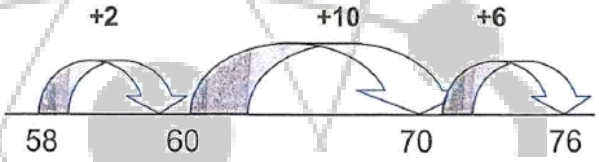
32- 28 = 4



'The difference between 28 and 32 is 4.'

If children are confident, further develop this method:

76 – 58 = 18



'The difference between 58 and 76 is 18.'

Further develop subtraction with numbers that bridge 100, using a **200 grid** to support.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

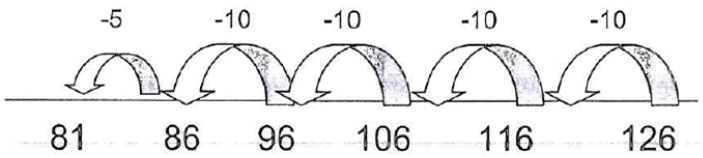
Subtraction – Year Three

Subtract numbers with up to three digits, using formal written method of columnar subtraction

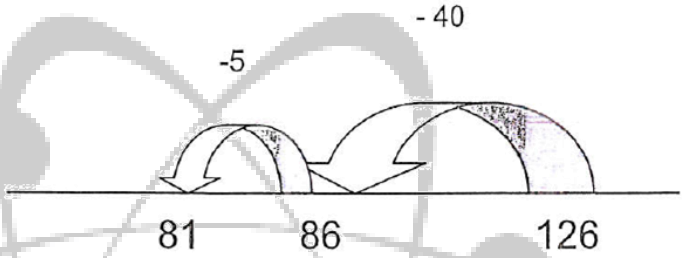
N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the use of the **empty number line** with calculations that **bridge 100**:

126 – 45 = 81

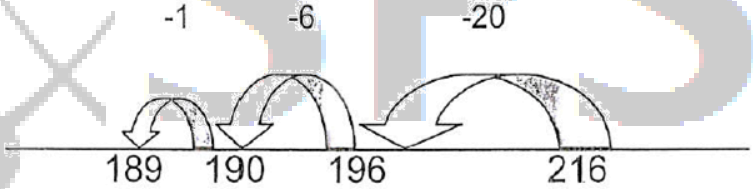


Then use more efficient jumps:



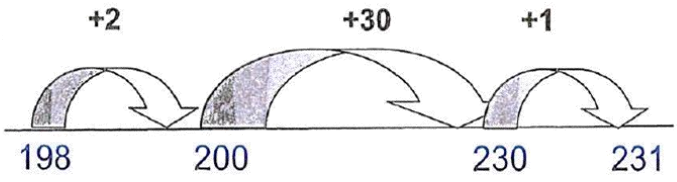
Extend with larger numbers by counting back:

216 – 27 = 189



… and by **counting on to find the difference** (small differences.)

231 – 198 = 33



'The difference between 198 and 231 is 33.'

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns). Use two-digit numbers when introducing this method, initially:

78 – 23 = 55

Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.

70 + 8
−20 + 3
50 + 5 = 55

N.B. In this example decomposition (exchange) is not required.

You might replace the **+ sign** with the word **'and'** to avoid confusion.

This will lead into the **formal written method:**

7 8
-2 3
5 5

Use the language of place value to ensure understanding.
'Eight subtract three, seventy subtract twenty.'

N.B. A number line would be an appropriate method for this calculation but use two-digit numbers to illustrate the formal written method initially.

Introduce the **expanded written method** where **exchange/decomposition** is required:

73 – 27 = 46

70 + 3 becomes 60 +13
- 20 + 7 - 20 + 7
40 + 6 = 46

73 is partitioned into 60 + 13 in order to calculate 73 – 27

N.B. Children will need to practise partitioning numbers in this way. Base ten materials could be used to support this.

When children are confident with the expanded method introduce the **formal written method,** involving decomposition/exchange:

73 – 27 = 46

Use the language of place value to ensure understanding.

'We cannot subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

$^{6}\cancel{7}\ ^{1}3$
- 2 7
4 6

Use base ten materials to support understanding.

If children are confident, extend the use of the **formal written method with numbers over 100,** returning to the expanded method first, if necessary.

235 – 127 = 108

$$\begin{array}{r} 2\,\overset{2}{\not{3}}\,\overset{1}{5} \\ -\ 1\,2\,7 \\ \hline 1\,0\,8 \end{array}$$

Use the language of place value to ensure understanding.

In this example it has only been necessary to exchange from the tens column.

Use base ten materials to support understanding.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtraction – Year 4

- **Subtract numbers with up to 4 digits using the formal written method of columnar subtraction <u>where appropriate</u>**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Continue to develop the **formal written method of subtraction** by revisiting the **expanded method** first, if necessary. Continue to use **base ten materials** to support understanding.

258 - 73 = 185

$$\begin{array}{r} 200 + 50 + 8 \\ -\quad 70 + 3 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 100 + 150 + 8 \\ -\quad 70 + 3 \\ \hline 100 + \quad 80 + 5 \end{array} = 185$$

You might replace the **+ sign** with the word **'and'** to avoid confusion. Children will need to practise partitioning in a variety of ways.

This leads to the **formal written method,** involving decomposition…

$$\begin{array}{r} 1\not{2}^{\,1}5\ 8 \\ -\ \ 7\ 3 \\ \hline 1\ 7\ 5 \end{array}$$

Use the language of place value to ensure understanding.

In this example it has been necessary to exchange from the hundreds column.

Further develop by subtracting a three-digit number from a three-digit number:

637 – 252 = 385

$$\begin{array}{r} 600 + 30 + 7 \\ -\ 200 + 50 + 2 \end{array} \qquad \begin{array}{r} 500 + 130 + 7 \\ -\ 200 + \ \ 50 + 2 \\ \hline 300 + \ \ 80 + 5 \end{array} \ = \ 385$$

Ensure that children are confident in partitioning numbers in this way.

This leads to a **formal written method:**

$$\begin{array}{r} {}^{5}\not{6}^{1}3\ 7 \\ -\ 2\ 5\ 2 \\ \hline 3\ 8\ 5 \end{array}$$

Use the language of place value to ensure understanding and use base ten materials, if necessary.

When children are confident, develop with **four digit numbers** and decimal numbers (in the context of money and measures.)

3625 – 1219 = 2406

$$\begin{array}{r} 3\ 6\ {}^{1}\not{2}^{15} \\ -\ 1\ 2\ 1\ 9 \\ \hline 2\ 4\ 0\ 6 \end{array}$$

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

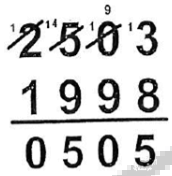
Year Five – Subtraction

- **Subtract whole numbers with more than 4 digits, including using the formal written method (columnar subtraction.)**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

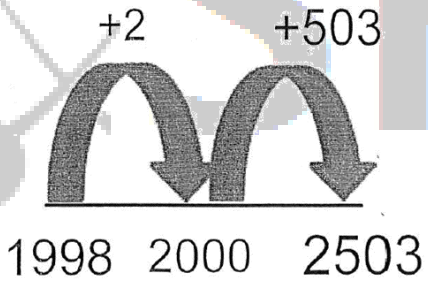
Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.

Continue to develop the formal written method for subtraction with three and four digit numbers (See Year 4 Guidance), returning to an expanded method and using base ten materials, if necessary.



There are no tens in the first number (2503) so we have to change a hundred for 10 tens before we can exchange a ten for ten units.

N.B. It would be appropriate to discuss the use of mental calculation methods with an example like this one, i.e. would an empty number line be a more efficient method for these numbers?



Introduce subtraction of decimals, initially in the context of money and measures.

Continue to practise and apply the formal written method with large numbers and decimals throughout Year 5.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Six – Subtraction

No objectives have been included in the programmes of study explicitly related to written methods of subtraction in Year 6. However, there is an expectation that children will continue to practice and sue the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance for methods.)

Our aim is that by the end of Year 6 children use mental **methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

Stages in Multiplication

Multiplication – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.

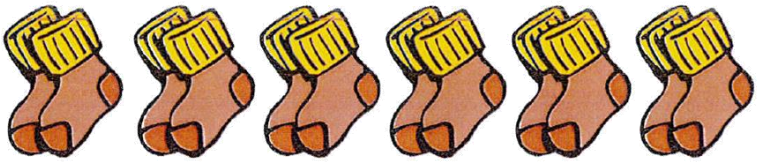
'Three apples for you and three apples for me. How many apples altogether?'

Multiplication – Year One

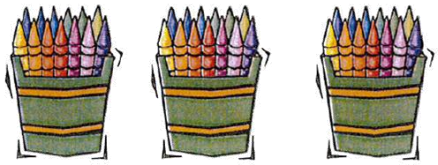
- **Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**

- **Count in multiples of twos, fives and tens (to the 10th multiple)**

Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts, They will solve **practical problems** that involve combining groups of 2, 5 or 10. e.g. socks, fingers and cubes.

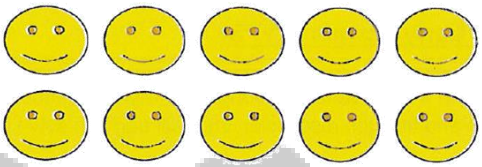


'Six pairs of socks.
How many socks altogether? 2, 4, 6, 8, 10, 12'



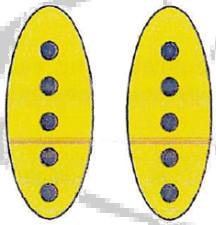
'Three pots of ten crayons, How many crayons altogether? 10, 20, 30'

Use arrays to support early multiplication



'Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10'
Two groups of five faces. How many faces altogether? 5, 10'

'2 groups of 5'



'How many altogether?'

'5 + 5 = 10'

Double five is ten

Continue to solve problems in **practical contexts** and develop the language of early multiplication, with appropriate resources, throughout Year 1.

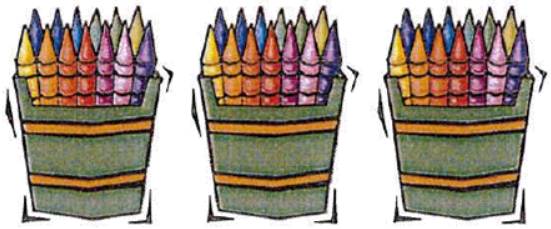
Multiplication – Year Two

- **Recall and use multiplication facts for the 2, 5 and 10 multiplication tables**
- **Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs**
- **Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in contexts**
- **Show that multiplication of two numbers can be done in any order (commutative)**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x sign to record.

Combining Groups (repeated addition):



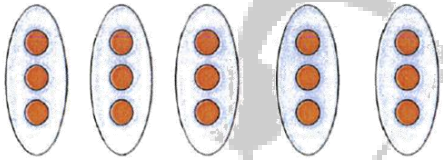
‘3 groups of 10 crayons’

‘How many crayons altogether?’

’10 + 10 + 10 = 30’

‘3 groups of 10’ ‘3 times ten

‘3 x 10 = 30’ ’10 x 3 = 30’



‘5 groups of 3’ ‘5 lots of 3’ ‘3 + 3 + 3 + 3 + 3 = 15’

‘5 times 3’ ‘3 multiplied by 5’ ‘5 x 3 = 15’ ‘3 x 5 = 15’

Using arrays to support multiplication:

6 x 5 = 30

‘5 + 5 + 5 + 5 + 5 + 5 = 30 ‘

‘6 rows of 5’

‘6 groups of 5’

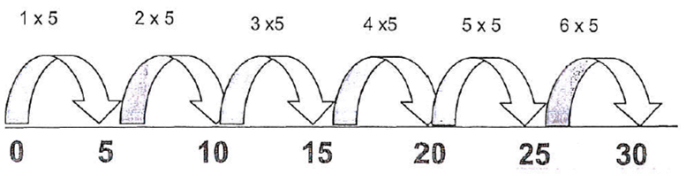
‘5 groups of 6’

‘5 x 6 = 30’

‘6 x 5 = 30’

Using an empty number line:

6 x 5 = 30



Make the link to repeated addition.

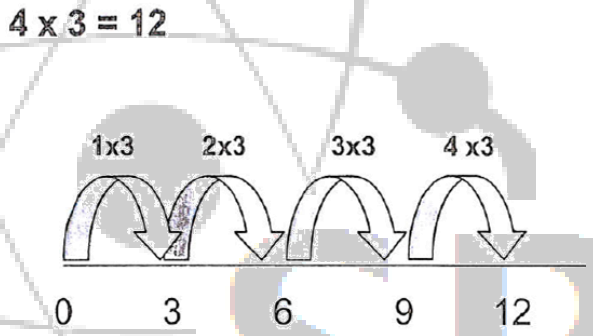
N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication – Year Three

- **Recall and use multiplication facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables
- **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including the two-digit numbers times one-digit numbers, <u>using mental methods</u> and progressing to a formal written method**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use **number lines** and **arrays** to support multiplication, as appropriate (see Year 2 guidance).



Partitioning method for multiplication of a teen number by a one-digit number:

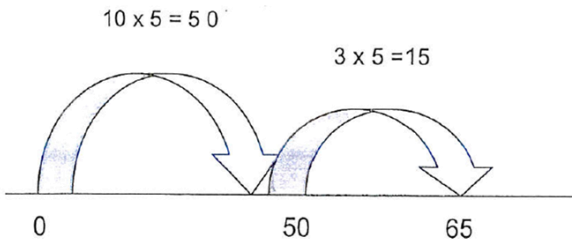
13 x 5 = 65 (Partition 13 into 10 + 3)

10 x 5 = 50
3 x 5 = 15

50 + 15 = 65

Demonstrate the partitioning method using a number line:

13 x 5 = 65



Grid Method (teen number multiplied by a one-digit number):

13 x 8 = 104

X	10	3
8	80	24

80 + 24 = 104

'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into **expanded short multiplication**:

13 x 8 = 104

Include an addition symbol when adding partial products.

```
10 + 3
x   8
  24  (3 x 8)
+ 80  (10 x 8)
 104
```

Refine the recording in preparation for formal short multiplication:

13 x 8 = 104

Use the language of place value to ensure understanding.

Include an addition symbol when adding partial products.

```
  13
x  8
  24  (3 x 8)
+ 80  (10 x 8)
 104
```

Model the same calculation using a number line, if necessary, to ensure understanding.

Formal short multiplication:

Ensure that the digit 'carried over' is written above the line in the correct column.

Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout Year 3 using teen numbers multiplied by a one-digit number.

If children are confident progress to multiplying other two-digit numbers by a one-digit number (see Year 4 guidance)

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication – Year Four

- **Recall multiplication facts for multiplication tables up to 12 x 12**
- **Multiply two-digit and three-digit numbers by a one-digit number using formal written layout**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use **empty number lines**, as appropriate (see Year 3 guidance)

Further develop the **grid method** for two-digit numbers multiplied by a one-digit number.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

Expanded short multiplication (two-digit number by a one-digit number):

36 x 4 = 144

```
 30 + 6
 X   4
   24    (4 x 6 = 24)
+ 120    (4 x 30 = 120)
  144
```

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication.

36 x 4 = 144

```
  36
 x  4
+ 24   (4 x 6)
 120   (4 x 30)
 144
```

This leads to **short multiplication (formal method)** of a two-digit number multiplied by a one-digit number.

36 x 4 = 144

```
  36
 x 4
 144
```

Use the language of place value to ensure understanding.

Ensure that the digit 'carried over' is written above the line in the correct column.

Continue to practise the formal method of short multiplication of a two-digit number by a one-digit number throughout Year 4.

If children are confident, continue to develop short multiplication with three-digit numbers multiplied by a one-digit number.

If necessary, return to the **grid method and/or expanded method** first:

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

This leads to expanded short multiplication:

127 x 6 = 762

```
  1 2 7
x   6
  4 2   (6x7)
+1 2 0   (6x20)
  6 0 0   (6x100)
  7 6 2
```

This will lead into short multiplication (formal method):

Use the language of place value to ensure understanding.

```
  1 2 7
x   6
  4 6
  7 6 2
```

Ensure that the digit 'carried over' is written above the line in the correct column.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication – Year Five

- **Multiply numbers up to 4 digits by a one – or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Build on the work covered in Year 4 with the formal method of short multiplication (two-digit number multiplied by a one-digit number).

When children are confident, introduce multiplication by a two-digit number. If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a teen-number):

23 x 13 = (20 + 3) x (10 + 3) = 299

X	20	3
10	200	30
3	60	9

```
  2 3 0
+   6 9
  2 9 9
```

Add the partial products (200 + 30) + (60 + 9) = 299

Expanded long multiplication (two-digit numbers multiplied by a teen-number):

23 x 13 = 299

```
  2 3
X 1 3
    9   (3 x 3)
  6 0   (3 x 20)
+ 3 0   (10 x 3)
2 0 0   (10 x 20)
2 9 9
```

This leads into…..

Compact long multiplication (formal method):

23 x 13 = 299

Use the language of place value to ensure understanding.

Add the partial products.

```
  2 3
X 1 3
+ 6 9   (3 x 23)
2 3 0   (10 x 23)
2 9 9
```

Extend to larger two-digit numbers:

56 x 27 = (50 + 6) x (20 + 7) = 1512

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Add the partial products (1000 + 120) + (350 + 42) = 1512

Expanded long multiplication (two-digit numbers multiplied by two-digit numbers)

56 x 27 = 1512 This expanded method is linked to the grid method

```
  5 6
X 2 7
  4 2   (7x6)
  3 5 0 (7x50)
+ 1 2 0 (20x6)
 1 0 0 0 (20x50)
 1 5 1 2
```

This leads into…..

Compact long multiplication (formal method):

56 x 27 = 1512

Use the language of place value to ensure understanding.

In this example there are digits that have been 'carried' over in the partial products.

```
   5 6
 x 2 7
   3 9 2   (7x56)
+ 1 1 2 0  (20x56)
 1 5 1 2
```

Add the partial products.

When children are confident with long multiplication extend wit three-digit numbers multiplied by a two-digit number, returning to the grid method first, if necessary.

Multiplication – Year Six

- **Multiply multi-digit numbers (including decimals) up to 4 digits by a two-digit whole number**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise and develop the **formal short multiplication** method and **formal long multiplication** method with larger numbers and decimals throughout Year 6. Return to an expanded form of calculation initially, if necessary (see Year 5 guidance.)

The **grid method** (decimal number multiplied by a two-digit number):

53·2 x 24 = 1276·8

x	50	3	0·2	
20	1000	60	4	1064·0
4	200	12	0·8	212·8
				1276·8

The formal written method of long multiplication:

```
  53·2
x  24·0
 212·8   (53·2 x 4)
1064·0   (53·2 x 20)
1276·8
```

It is an option to include .0 in this example, but not essential.

The prompts (in brackets) can be omitted of children no longer need them.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Year 6 children use mental methods (with jottings) when appropriate, but for calculations they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

Stages in Division

Division – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people.

'Half of the apples for you and half of the apples for me.'

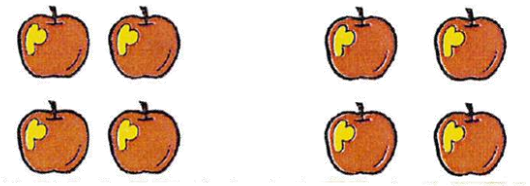
Division -Year One

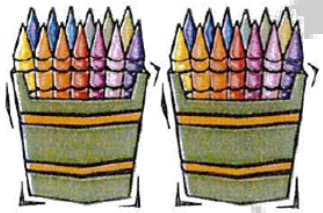
- **Solve one-step problems involving division be calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**
- **Count in multiples of twos, fives and tens (to the 10th multiple)**

Children will start with practical **sharing** using a variety of resources.

They will share objects into **equal groups** in a variety of situations.
They will begin to use the vocabulary associated with division in practical contexts.

'Share these eight apples equally between two children. How many apples will each child have?'

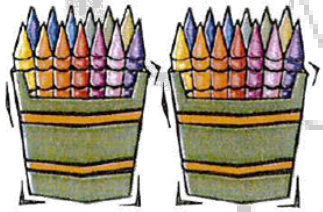




'Share 20 crayons between 2 pots.'

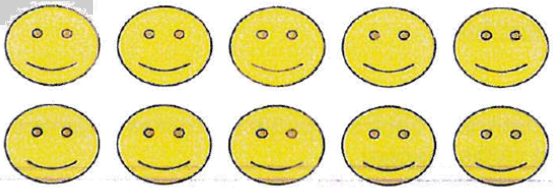
'How many crayons are in each pot?'

Children will move from **sharing** to **grouping** in a practical way.

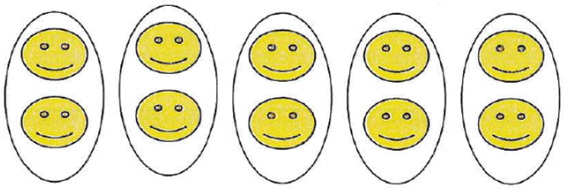


'Put 20 crayons into groups of 10. How many pots do we need?'

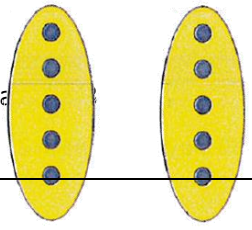
Use **arrays** to support early division



'How many faces altogether? How many groups of two?'



'Five groups of two'



'How many groups of 5?'

Pa

’10 shared equally between 2 people’
‘Half of ten is five’

Continue to solve problems in **practical contexts** throughout Year 1, and develop the language of early division, with appropriate resources.

Division – Year Two

- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables**
- **Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs.**
- **Solve problems involving division, using materials, arrays, repeated subtraction, mental methods and multiplication and division facts, including problems in contexts.**

N.B. Ensure that children are confident with the methods outlined in the previous year’s guidance before moving on.

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the ÷ sign to record, using multiples that they know,

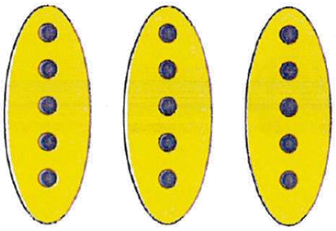
Sharing and grouping:

’30 crayons shared equally between 3 pots.’ (Sharing)

‘We have 30 crayons and put 10 crayons in each pot, how many pots do we need?’ (Grouping)

’30 divided by 10 = 3’
’30 divided by 3 = 10’

30 ÷ 3 = 10
30 ÷ 10 = 3



‘How many groups of 5?’
’15 shared equally between 3 people is…?’

’15 divided by 3 equals 5’
’15 divided by 5 equals 3’

15 ÷ 5 = 3
15 ÷ 3 = 5

Using arrays to support division

15 ÷ 5 = 3
15 ÷ 3= 5

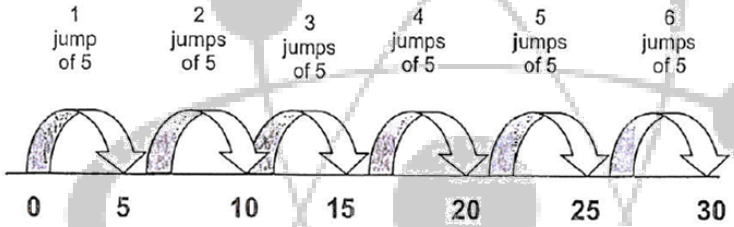
How many groups of 3?
How many groups of 5?
15 shared between 3 people is…?
15 shared between 5 people is…?

15 divided by 5 = 3
15 divided by 3 = 5

When children are ready, use an empty number line to count forwards:

30 ÷ 5 = 6

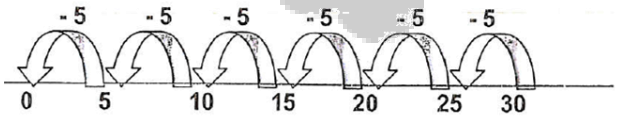
'How many jumps of five make thirty?'



Also jump back to make the link with repeated subtraction:

30 ÷ 5 = 6

'How many groups of five?'



N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

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Year Three – Division

- **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
- **Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method.**

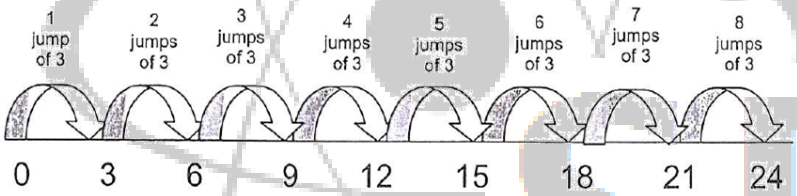
N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the ÷ sign to record, using multiples that they know, as appropriate (see Year 2 guidance.)

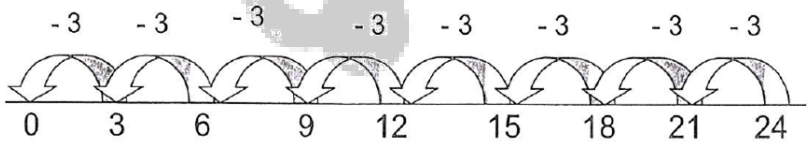
Using an empty number line to count forwards:

24 ÷ 3 = 8

'How many threes in 24?'



…also jump back from 24 to make the link with repeated subtraction



'How many groups of 3 in 24?'

Introduce the formal layout using multiplication/division facts that the children know:

24 ÷ 3 = 8

This can also be recorded as…

$$3\overline{)\,24}\quad 8$$

'Twenty four divided by three equals eight.'

'How many threes are there in twenty four?'

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Four – Division

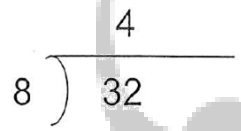
- **Recall multiplication and division facts for multiplication tables up to 12 x 12**
- **Use place value, known and derived facts to divide mentally**
- **Divide two-digit and three-digit numbers by a one-digit number using formal written layout** (not explicitly stated in the programme of study but implied in the non-statutory guidance.)

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know, e.g.

32 ÷ 8 = 4

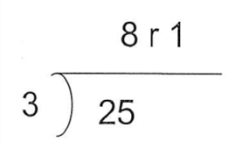
Continue using the formal written layout for division using the multiplication tables that they know:



'How many eights are there in thirty two?'

Continue using the formal written layout, introducing remainders:

25 ÷ 3 = 8 r1

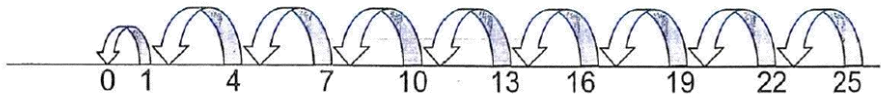


N.B. Remainders are not specifically referred to until Year 5 in the National Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts.

This could be modelled using an empty number line, if necessary:

'Eight jumps of three and one left over.'

25 ÷ 3 = 8 r1



Alternatively you could jump forwards in multiples of three from zero to twenty four ('and one more makes 25')

Division using partitioning (two digits divided by one digit):

65 ÷ 5 = 13

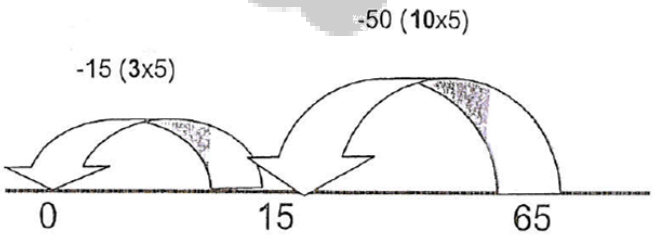
65 = 50 + 15

50 ÷ 5 = 10
15 ÷ 5 = 3
10 + 3 = 13

N.B. Children will need to practice partitioning in a variety of ways.

Continue to use **empty number lines**, as appropriate, using multiples of the divisor:

65 ÷ 5 = 13



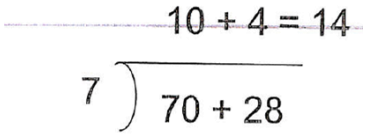
98 ÷7 = 14

98 = 70 + 28 Partition 98 into 70 and 28
70 ÷7 = 10
28 ÷7 = 4
10 + 4 = 14

This could be modelled on an empty number line to further develop understanding.

N.B. Children will need to practise partitioning in a variety of ways.

98 ÷ 7 = 14

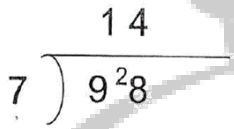


'We have partitioned 98 into 70 and 28 (90 = 70 + 28)

Seven 'goes into' 70 ten times and seven 'goes into' 28 four times.
Ten add four equals 14.'

This will lead into the formal method of short division:

98 ÷ 7 = 14



Use the vocabulary of place value to ensure understanding and make the link to partitioning.

Continue to practise the formal method of short division throughout Year 4.

If children are confident develop further, by dividing three-digit numbers by a one-digit number using the formal method of short division with whole number answers (no remainders).

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

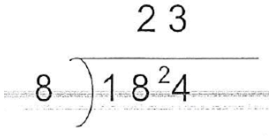
Year 5 – Division

- **Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise the formal written method of short division with whole number answers…

184 ÷ 8 = 23



Use the vocabulary of place value to ensure understanding.

Make the link to the partitioning method (see Year 4 guidance).

….and with remainders:

$$432 \div 5 = 86 \text{ r} 2$$

$$5 \overline{)\, 4\,3^{3}2} \quad 8\,6\,\text{r}\,2$$

The remainder can be expressed as a fraction, $\frac{2}{5}$ (the remainder divided by the divisor):

$$432 \div 5 = 86\tfrac{2}{5}$$

Continue to practise, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the context.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Year 6 – Division

- **Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context**
- **Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.**

N.B. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

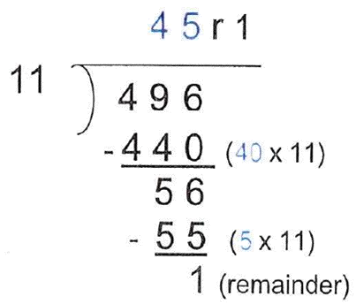
Continue to practise the **formal method of short division**, with and without remainders using the language of place value to ensure understanding (see Year 5 guidance).

$$496 \div 11 = 45 \text{ r} 1$$

$$11 \overline{)\, 4\,9^{5}6} \quad 4\,5\,\text{r}\,1$$

The remainder can also be expressed as a fraction, $\frac{1}{11}$ (the remainder divided by the divisor).

Dividing by a two-digit number using a **formal method of long division**:



Multiples of the divisor (11) have been subtracted from the dividend (496)

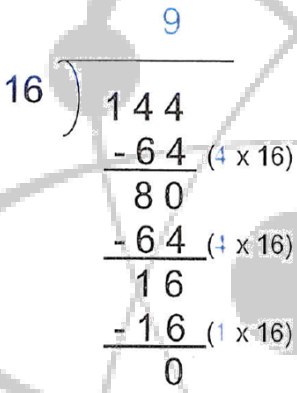
'40 (lots of 11) + 5 (lots of 11) = 45 (lots of 11)'

'1 is the remainder'

Answer : $45\frac{1}{11}$

Standard short division does not help with the following calculation. However it can be solved using **long division** (by repeated subtraction using multiples of the divisor):

144÷ 16 = 9



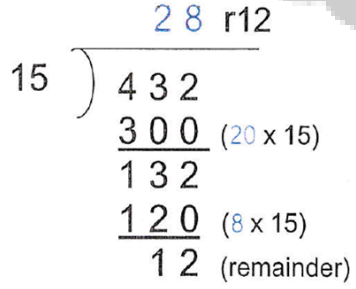
Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16)

There is no remainder.'

Children will need to select the most effective method for each calculation/problem they meet, including whether to use the standard, **formal written method of long division**:

432 ÷15 = 28 r2



Multiples of the divisor (215) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28 (lots of 15)

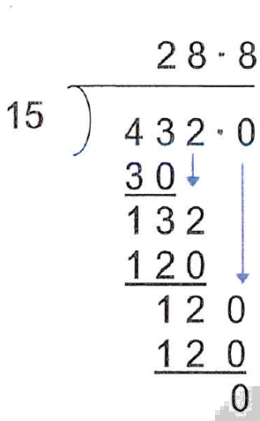
12 is the remainder'

The remainder can also be expressed as a fraction, $\frac{12}{15}$ (the remainder divided by the divisor) or as a decimal, 0.8 (see next example)

The answer is: $28\frac{12}{15}$ or 28.8

This is an alternative way of recording formal long division:

$432 \div 15 = 28{\cdot}8$



N.B. Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.

N.B. If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Year 6 children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.